Prelim - Model Answer Paper

Geometry

Time : 2½ Hrs. **(Pages 14)** **Marks : 60**

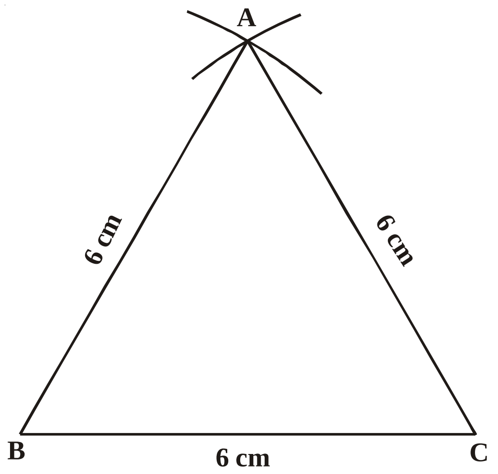
A.1. **Attempt any six of the following sub-questions :**

(i) $\Delta DEF \sim \Delta MNK$ [Given] **1**

$\therefore \dfrac{A(\Delta DEF)}{A(\Delta MNK)} = \dfrac{DE^2}{MN^2} = \dfrac{5^2}{6^2}$ [Areas of similar triangle]

$\therefore \dfrac{A(\Delta DEF)}{A(\Delta MNK)} = \dfrac{5 \times 5}{6 \times 6} = \dfrac{25}{36}$

(ii) **1**



(iii) **Given :** Side of a Cube = 60 cm **1**

To find : Total surface area of a cube

Total surface area of a cube $= 6 \times \text{side}^2$

$= 6 \times (60)^2$

$= 6 \times 3600$

$= 21600 \text{ cm}^2$

$\therefore$ **Total surface area of cube is 21600 cm²**

(iv) $y = 2x - 3$ **1**

For getting x-intercept, put $y = 0$

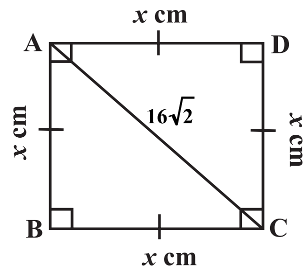
$y = 2x - 3$

$0 = 2x - 3$

$2x = 3$

	∴ $x = \frac{3}{2}$	
	For getting y-intercept, put $x = 0$	
	$y = 2x - 3$	
	$y = 2(0) - 3$	
	∴ $y = 0 - 3$	
	$y = -3$	
	∴ **x-intercept = $\mathbf{\frac{3}{2}}$, y-intercept = –3**	
(v)	$\Delta APQ \sim \Delta ABC$, [Given]	1
	$\frac{AP}{AB} = \frac{AQ}{AC}$ [c.s.s.t]	
	∴ $\frac{6}{15} = \frac{4}{AC}$	
	∴ **AC = 10 units.**	
(vi)	$\sin\theta = \frac{7}{5}$ [Given]	1
	We know,	
	∴ $\text{cosec}\theta = \frac{1}{\sin\theta}$	
	∴ $\text{cosec}\theta = \frac{1}{\frac{7}{5}}$	
	∴ **$\mathbf{cosec\theta = \frac{5}{7}}$**	
(vii)	$\frac{\tan 49^\circ}{\tan 49^\circ} = \frac{\tan 49^\circ}{\tan(90 - 41^\circ)}$	1
	∴ $\mathbf{\frac{\tan 49^\circ}{\tan 49^\circ} = 1}$	
A.2.	**Attempt any five of the following sub-questions :**	
(i)	❑ABCD is a square.	
	∴ $\angle ABC = 90^\circ$ ...(i) [Angle of a square]	1
	$AC = 16\sqrt{2}$ cm [Given]	

Let the length of each side of square be x cm.



Now in ΔABC,

$\angle ABC = 90^\circ$ [From (i)]

$\therefore \quad AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\therefore \quad \left(16\sqrt{2}\right)^2 = x^2 + x^2$

$\therefore \quad 256 \times 2 = 2x^2$

$\therefore \quad x^2 = \dfrac{256 \times 2}{2}$

$\therefore \quad x^2 = 256$

$\therefore \quad x = 16$ [Taking the square root]

$\therefore$ **Length of the side of a square is 16 cm.** **1**

(ii) **Analysis :**

Chord MN subtends an $\angle MPN$.

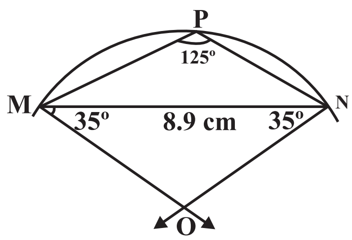
$\angle MPN$ is inscribed in arc MPN.

$\angle NMO = \angle MNO$

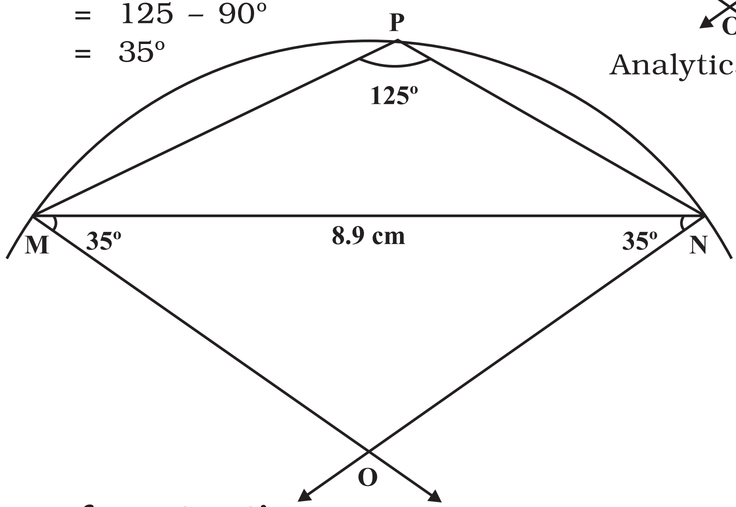
$= \angle MPN - 90^\circ$

$= 125 - 90^\circ$

$= 35^\circ$



Analytical Figure



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Steps of construction :

1. Draw seg MN of length 8.9 cm. Draw ray MO and NO making an angle 35° (125° – 90°) each below the chord on same side. Point O is the intersecting point of these rays.
2. Draw an arc with centre 'O' and radius OM or ON on the centre side.
3. Take any point P on the arc. Join MP and NP. Arc MPN is the required arc. Measure of inscribed angle is 125°.

(iii)	If $\sin\theta = \frac{5}{13}$, θ is the acute angle.	
	$\sin^2 + \cos^2 = 1$	
	$\therefore \quad \cos^2 = 1 - \sin^2\theta - 1$	**1**
	$\therefore \quad \cos^2\theta = 1 - \left[\frac{5}{13}\right]^2$	
	$\therefore \quad \cos^2\theta = 1 - \frac{25}{169}$	
	$\therefore \quad \cos^2\theta = \frac{169 - 25}{169}$	
	$\therefore \quad \cos^2\theta = \frac{144}{169}$	
	$\therefore \quad \mathbf{\cos^2\theta = \frac{12}{13}}$ [Taking the square root]	**1**
(iv)	Let P(–3, 11), Q(6, 2) and R(k, 4) are the given points. $\downarrow \quad \downarrow \quad \downarrow$ $(x_1, y_1) \quad (x_2, y_2) \quad (x_3, y_3)$	
	Points P, Q, R are collinear. [Given]	
	$\therefore$ Slope of line PQ = Slope of line QR	
	$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$	**1**
	$\therefore \quad \frac{2 - 11}{6 - (-3)} = \frac{4 - 2}{k - 6}$	
	$\therefore \quad \frac{-9}{6 + 3} = \frac{2}{k - 6}$	
	$\therefore \quad \frac{-9}{9} = \frac{2}{k - 6}$	
	$\therefore \quad -1 = \frac{2}{k - 6}$	
	$\therefore \quad -k + 6 = 2$	
	$\therefore \quad k = 6 - 2$	
	$\therefore \quad \mathbf{k = 4}$	**1**
(v)	**Given** : breadth (b) = 5cm, height (h) = 4cm and total surface area = 166cms^2	
	To find : length of the cuboid (l) = ?	

Total surface area of a cuboid = $2(lb + bh + lh)$ **1**

∴ $166 = 2(l \times 5 + 5 \times 4 + l \times 4)$

∴ $166 = 2(9l + 20)$

∴ $l = 7$

∴ **Length of the cuboid is 7cm.** **1**

(vi) Volume of cube = $0.027\ cm^3$ **1**

∴ $l^3 = 0.027$

∴ $l^3 = \frac{27}{100}$

∴ $l = \frac{3}{10}$

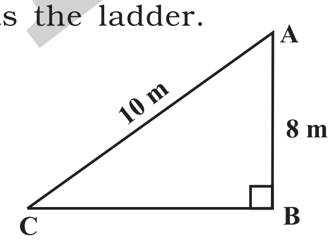
∴ **Length of sides of a cube is 0.3 cm.** **1**

A.3. Attempt any four of the following sub-questions :

(i) In adjoining figure,

seg AB represents the wall.

A is the window and seg AC represents the ladder.



AB = 8 m, AC = 10 m [Given]

In ΔABC,

$m\angle ABC = 90°$ **1**

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

∴ $(10)^2 = (8)^2 + BC^2$ [Substituting the given values]

∴ $100 = 64 + BC^2$

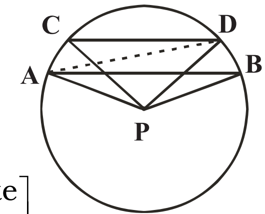
∴ $BC^2 = 100 - 64$ **1**

∴ $BC^2 = 36$

∴ $BC = 6m$ [Taking the square root]

∴ **The distance of the foot of the ladder from the base of the wall is 6 m.** **1**

(ii) **Construction :** Draw seg AD.



Proof :

Chord CD || Chord AB [Given]

On transversal AD,

$\angle CDA \cong \angle BAD$ ...(i) [Converse of alternate angles test] **1**

	$\angle CDA = \frac{1}{2}\, m\,(\text{arc AC})$ ...(ii) } [Inscribed angle Theorem]	
	$\angle BAD = \frac{1}{2}\, m\,(\text{arc BD})$ ...(iii) }	
	$\therefore \frac{1}{2}\, m(\text{arc AC}) = \frac{1}{2}\, m(\text{arc BD})$ [From (i), (ii) and (iii)]	1
	$\therefore m\,(\text{arc AC}) = m(\text{arc BD})$ ...(iv)	
	$\angle CPA = m\,(\text{arc AC})$ ...(v) [Definition of measure of minor arc]	
	$\angle DPB = m\,(\text{arc BD})$ ...(vi)	
	$\therefore$ **$\angle CPA = \angle DPB$** [From (iv), (v) and (vi)]	1
(iii)	sinθ A, B, C are the angle of ∠ABC	
	$\therefore A + B + C = 180^\circ$	1
	$B + C = 180 - A$	
	$\therefore \frac{B + C}{2} = \frac{180 - A}{2}$ [Dividing throughout by 2]	
	$\therefore \frac{B + C}{2} = \frac{180}{2} - \frac{A}{2}$	1
	$\frac{B + C}{2} = 90 - \frac{A}{2}$	
	$\sin\left(\frac{B + C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$	
	$\mathbf{\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}}$ [$\because$ sin (90 – θ) = cosθ]	1
(iv)	$\frac{x}{2} + \frac{y}{3} = 1$	
	$\therefore \frac{3x + 2y}{6} = 1$	1
	$\therefore 3x + 2y = 6$	
	$\therefore 2y = -3x + 6$	
	$y = \frac{-3}{2}x + \frac{6}{2}$ [Dividing both sides by 2]	1
	$y = \frac{-3}{2}x + 3$	
	$\therefore$ **$y = \frac{-3}{2}x + 3$ is the required equation which is in $y = mx + c$ form.**	1

(v) **Given :** Surface area of the sphere (S) = 616 cm^2

To find : Volume of a sphere

Surface area of the sphere = $4\pi r^2$ ½

$$616 = 4 \times \frac{22}{7} \times r^2$$

$$\therefore \quad r^2 = \frac{616 \times 7}{4 \times 22}$$ ½

$$r^2 = 49$$

$$r = 7 \qquad \text{[Taking square roots]}$$

Volume of the sphere = $\frac{4}{3}\pi r^3$ 1

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{4312}{3}$$

$$= 1437.35 \text{ cm}^3$$

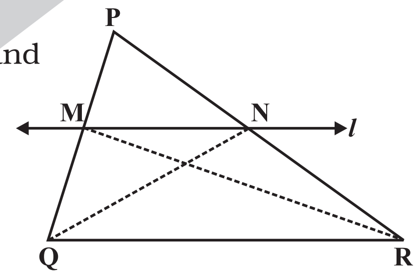
$\therefore$ **Volume of the sphere is 1437.35 cm^3** 1

A.4. Attempt any three of the following sub-questions:

(i) **Given :** In ΔPQR, line $l \parallel$ side QR

Line l intersects side PQ and side PR in points M and N respectively.

P-M-Q and P-N-R



To Prove : $\frac{PM}{MQ} = \frac{PN}{NR}$

Construction: Join seg QN and seg RM 1

Proof : In ΔPMN and ΔQMN,

$$\frac{A(\Delta PMN)}{A(\Delta QMN)} = \frac{PM}{QM} \quad \text{...(i)} \quad \text{[Triangles having equal heights]}$$ 1

In ΔPMN and ΔRMN,

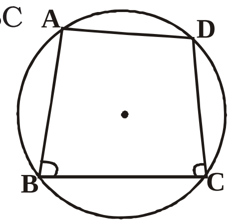
$$\frac{A(\Delta PMN)}{A(\Delta RMN)} = \frac{PN}{RN} \quad \text{...(ii)} \quad \text{[Triangles having equal heights]}$$

$A(\Delta QMN) = A(\Delta RMN)$ ...(iii) [Triangles with common base MN and same height] 1

$$\therefore \quad \frac{A(\Delta PMN)}{A(\Delta QMN)} = \frac{A(\Delta PMN)}{A(\Delta RMN)} \quad \text{[From (i), (ii) and (iii)]}$$

$$\therefore \quad \mathbf{\frac{PM}{MQ} = \frac{PN}{NR}}$$ 1

(ii) **To Prove :** side DC $\cong$ side AB, side AD $\parallel$ side BC



Proof :

$\angle ABC \cong \angle BCD$ ...(i) [Given]

$\angle ABC = \frac{1}{2} m(\text{arc ADC})$ ...(ii)

[Inscribed Angle theorem] **1**

$\angle BCD = \frac{1}{2} m(\text{arc DAB})$ ...(iii) [Inscribed Angle theorem]

$\therefore \frac{1}{2} m(\text{arc ADC}) = \frac{1}{2} m(\text{arc DAB})$ [From (i), (ii), (iii)] **1**

$\therefore$ m (arc ADC) = m (arc DAB)

$\therefore$ m (arc AD) + m(arc DC) = m(arc AD) + m(arc AB)

[Arc Addition Property] **1**

$\therefore$ m(arc DC) = m(arc AB)

$\therefore$ arc DC $\cong$ arc AB

$\therefore$ chord DC $\cong$ chord AB [In the same circle congruent arcs have their chords congruent]

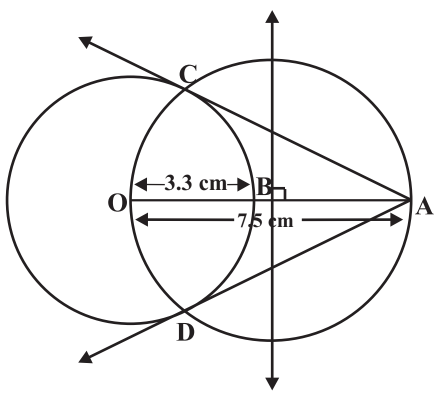
i.e side AB $\cong$ side DC

$\angle ADC + \angle ABC = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary]

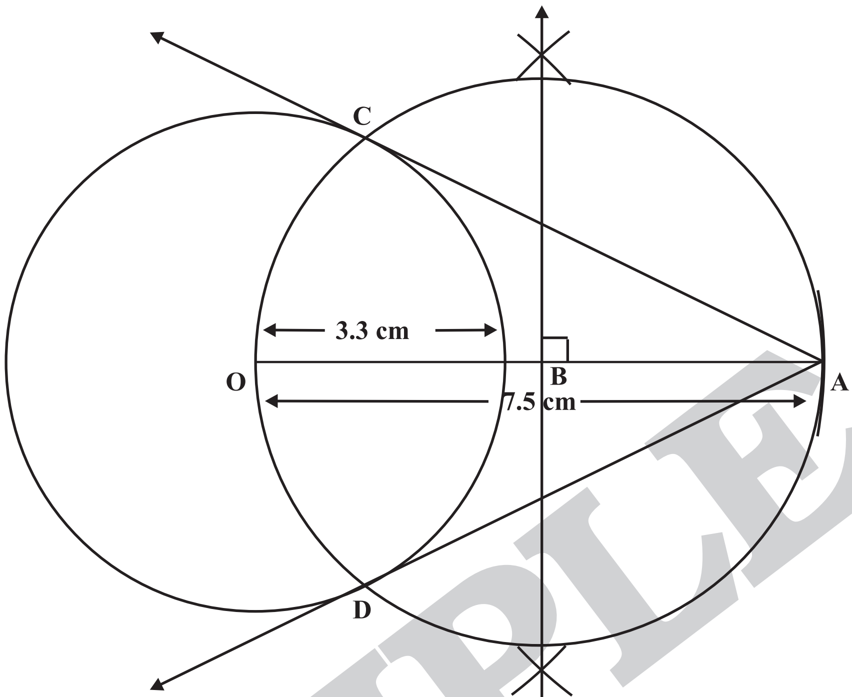
$\therefore$ $\angle ADC + \angle BCD = 180°$ [From (i)]

$\therefore$ **side AD || side BC** [Interior Angles Test] **1**

(iii)



Analytical figure



$l(AC) = l(AD) = 6.7$ cm **1**

Steps of construction :

1. Construct a circle with centre O and radius 3.3 cm. Take point A such that OA = 7.5 cm. **1**
2. Obtain midpoint B of seg OA. Draw a circle with centre B and radius BA. **1**
3. Let D and C be the points of intersection of these two circles. Draw lines AC and AD which are the required tangents. **1**

(iv) **Given :** A cone and hemisphere of equal bases and volume

To find : $h_{cone} : h_{hemisphere}$

Volume of cone = Volume of hemisphere **1**

$$\frac{1}{3} \times \text{Area of base} \times h_c = \frac{2}{3} \times \text{Area of base} \times h_h$$ **1**

But Area of base of cone = Area of base of hemisphere **1**

$$\therefore \quad \frac{h_c}{3} = \frac{2h_h}{3}$$

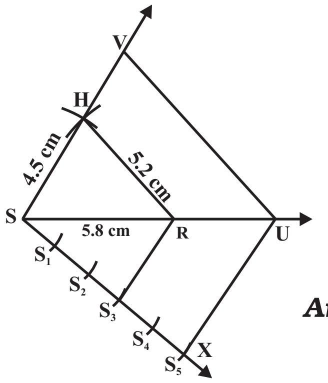
$$hc = 2h_h$$

$$\therefore \quad \frac{h_c}{h_h} = \frac{2}{1}$$

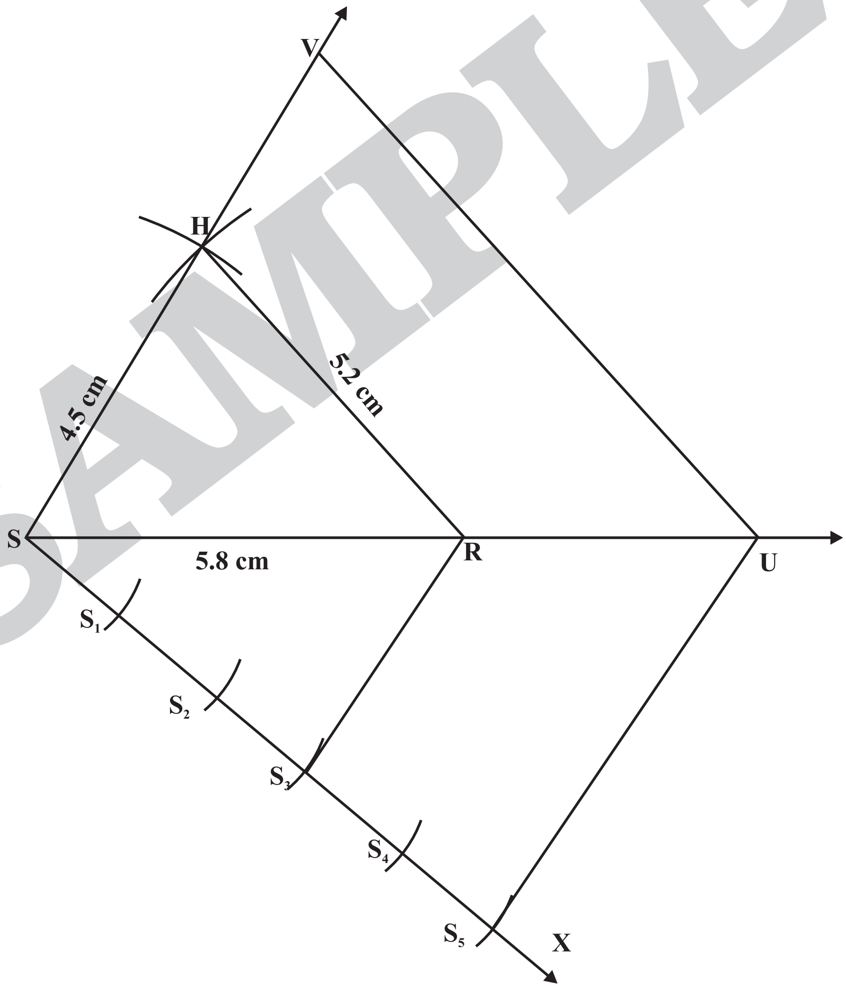
$\therefore$ **The ratio of the height of cone and a hemisphere is 2:1.** **1**

A.5. **Attempt any four sub-questions of the following :**

(i)



Analytical figure

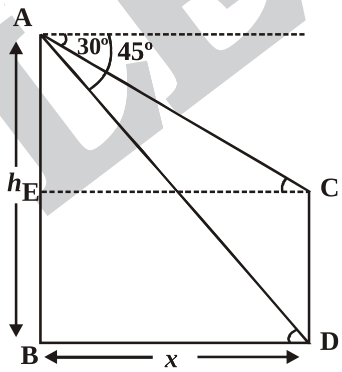


Steps of construction :

1. Construct ΔSRH with SR = 5.8 cm, SH = 4.5 cm and HR = 5.2 cm. **1**
2. Below SR, make an acute angle $\angle RSX$. **1**
3. Along SX, mark five points S_1, S_2, S_3, S_4, S_5 such that $SS_1 = S_1S_2 = S_2S_3 = S_3S_4 = S_4S_5$. **1**
4. Since we have to construct a triangle each of whose sides is 5/3 of the corresponding sides of ΔSRH. So Join S_3R and draw line A_5U parallel to S_3R. **1**
5. Draw a line through U parallel to RH intersecting the extended line SH at V. **1**

$\therefore$ **ΔVSU is the required triangle.**

(ii) Let AB be the lighthouse and CD be the ship. Let the height of the lighthouse be h and let the distance from the ship to the lighthouse is x. Given, the height of the ship is 24m. the angle of depression to the top of the mast is 30° and the angle of depression to the base of the ship is 45°, CE $\perp$ AB



$$\tan 45^\circ = \frac{h}{x}$$

But $\tan 45^\circ = 1$, $\therefore$ $x = h$ ...(i) **1**

$$\tan 30^\circ = \frac{AE}{EC} = \frac{h-24}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h-24}{x}$$

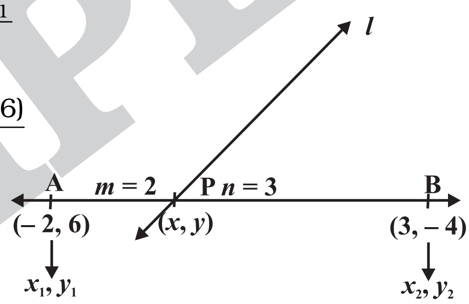
$$h - 24 = \frac{x}{\sqrt{3}} \quad \text{...(ii)}$$ **1**

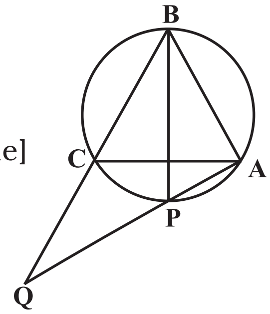
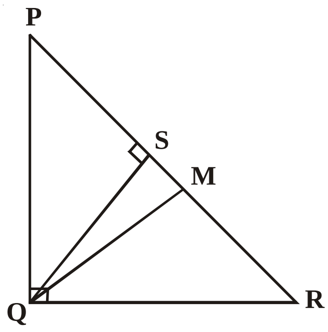
From equation (i) and (ii), we get;

$$x - 24 = \frac{x}{\sqrt{3}}$$

$$x - \frac{x}{\sqrt{3}} = 24$$

$$(\sqrt{3} - 1)x = 24\sqrt{3}$$

	$x = \dfrac{24\sqrt{3}}{(\sqrt{3}-1)}$	1
	$x = \dfrac{24\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3}-1)\,(\sqrt{3}+1)}$ [Rationalizing denominator]	
	$= \dfrac{24\,(3+\sqrt{3})}{(3-1)}$	
	or $x = 12\,(3+\sqrt{3}) = 12\,(3+1.73) = 12 \times 4.73 = 56.76$	
	∴ **The ship is at a distance of 56.76m from the lighthouse.**	1
(iii)	Let l be the required line.	
	By section formula - internal division we find	
	the co-ordinate of P(x, y)	
	$x = \dfrac{mx_2 + nx_1}{m+n}$, $y = \dfrac{my_2 + ny_1}{m+n}$ 	
	∴ $x = \dfrac{2(3)+3(-2)}{2+3}$, $y = \dfrac{2(-4)+3(6)}{2+3}$	
	∴ $x = 0$, $y = \dfrac{10}{5} = 2$	1
	∴ P ≡ (0, 2)	
	The line 'l' passes through the point P(0, 2) = (x_1, y_1) and has	
	slope $= \dfrac{3}{2}(m)$	1
	∴ The equation of line l by point slope form is	
	$y - y_1 = m(x - x_1)$	1
	∴ $y - 2 = \dfrac{3}{2}(x - 0)$	
	∴ $2(y - 2) = 3x$	1
	∴ $2y - 4 = 3x$	
	∴ **$3x - 2y + 4 = 0$ is the required equation.**	1

(iv)	**Proof :**	
		
	In ΔABC	
	Side AB $\cong$ Side AC [Sides of an equilateral triangle]	
	$\therefore$ $\angle ABC \cong \angle ACB$ ...(i) [Isosceles triangle theorem]	1
	BP is the bisector of $\angle B$ [Given]	
	$2\angle PBC = \angle ABC$ ...(ii)	1
	In ΔAQC	
	$\angle ACB = \angle CAQ + \angle AQC$ [Remote Interior angle theorem]	
	$\angle ABC = \angle CAQ + \angle AQC$ [From (i)]	
	$\therefore$ $2\angle PBC = \angle CAQ + \angle AQC$ [From (ii)]	1
	But $\angle PBC \cong \angle CAP$ ...(iii) [Angle inscribed in same arc are congruent]	
	$\angle PBC \cong \angle CAQ$ [A-P-Q]	1
	$\therefore$ $2\angle PBC = \angle PBC + \angle AQC$	
	$\angle PBC = \angle AQC$	
	$\therefore$ $\angle CAP = \angle AQC$ [From (iii)]	
	$\therefore$ $\angle CAQ = \angle AQC$ [$\because$ A-P-Q]	
	$\therefore$ **CQ = CA** **[Isosceles triangle theorem]**	1
(v)	In ΔPQR, seg QM is bisector of $\angle PQR$	
		
	$\therefore$ $\frac{PQ}{QR} = \frac{PM}{MR}$ [Property of an angle bisector of a triangle]	
	$\therefore$ $\frac{PQ^2}{QR^2} = \frac{PM^2}{MR^2}$ ...(i) [Squaring both sides]	1
	In ΔPQR, $m\angle PQR = 90°$, seg QS $\perp$ seg PR [Given]	
	$\therefore$ $\Delta PSQ \sim \Delta QSR$ [Similarity of right angled triangle]	
	$\therefore$ $\frac{PS}{QS} = \frac{QS}{SR} = \frac{PQ}{QR}$ ...(ii) [c.s.c.t]	1

Consider,

$$\frac{PS}{QS} = \frac{QS}{SR}$$

$\therefore$ $QS^2 = PS \times SR$ ...(iii) **1**

Also, $\frac{PS}{QS} = \frac{PQ}{QR}$ [From (ii)]

$\therefore$ $\frac{PS^2}{QS^2} = \frac{PQ^2}{QR^2}$ [Squaring both]

But $QS^2 = PS \times SR$ [From (iii)]

$\therefore$ $\frac{PS^2}{PS \times SR} = \frac{PQ^2}{QR^2}$

$\therefore$ $\frac{PS}{SR} = \frac{PQ^2}{QP^2}$ ...(iv) **1**

$\therefore$ $\frac{PM^2}{MR^2} = \frac{PQ^2}{QR^2}$ [From (i), (iv)] **1**

$\therefore$ $\mathbf{\frac{PS}{SR} = \frac{PM^2}{NR^2}}$

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