Prelime - Model Answer Paper

Algebra

Time : 2½ Hrs. **(Pages 14)** **Marks : 60**

A.1. Attempt any six of the following subquestions :

(i) We have $S_n = \frac{n}{2} \times [2a + (n - 1)d]$

Here $a = 7$, $d = 4$ and $n = 60$

$\therefore \quad S_{60} = \frac{60}{2} \times [2 \times 7 + (60 - 1) \times 4]$

$= 30[14 + 59 \times 4]$

$= 30[14 + 236]$

$\therefore \quad = 30 \times 250$

$\therefore \quad \mathbf{S_{60} = 7500}$ — **1**

(ii) Here $a = 10$, $d = -2$, $n = 5$

We know that, $t_n = a + (n - 1)d$

$t_5 = 10 + (5 - 1)(-2)$

$= 10 + 4 \times -2$

$= 10 - 8$

$\therefore \quad \boldsymbol{t_5 = 2}$ — **1**

(iii) $\frac{x^2}{3} = -1$

$\therefore \quad x^2 = -3$

$\therefore \quad x^2 - 3 = 0$

$\therefore \quad \boldsymbol{x^2 + 0x - 3 = 0}$ — **1**

(iv) $3x^2 - kx - 1 = 0$

Substituting $x = 1$, we get;

$x = 1$ is the root of the given equation so, it satisfies the given equation.

$\therefore \quad 3(1)^2 - k(1) - 1 = 0$

$\therefore \quad 3 \times 1 - k - 1 = 0$

$\therefore \quad 3 - k - 1 = 0$

$\therefore \quad -k + 2 = 0$

$\therefore \quad -k = -2$

$\therefore \quad \boldsymbol{k = 2}$ — **1**

(v)	$3x + 6y = 5$ ... (i) $6x + 3y = 4$ ... (ii) Adding eq.(i) and eq (ii), we get; $3x + 6y = 5$ $6x + 3y = 4$ $9x + 9y = 9$ $\therefore$ $9(x + y) = 9$ $\therefore$ $\boldsymbol{x + y = 1}$	1
A.2.	**Attempt any five of the following subquestions :**	
(i)	The given sequence is an A.P. with $a = 5$, $d = 3$ and $t_n = 68$ We know that, $t_n = a + (n - 1)d$ $\therefore$ $68 = 5 + (n - 1) \times 3$ $\therefore$ $68 = 5 + 3n - 3$ $\therefore$ $3n = 66$ $\therefore$ $n = 22$ $\therefore$ **22th term of the sequence is 68.**	1 1
(ii)	0 and –4 The roots of the quadratic equation are 0 and –4 Let $\alpha = 0$ and $\beta = -4$ $\therefore$ $\alpha + \beta = 0 + (-4)$ and $\alpha\beta = 0 \times (-4)$ $= 0 - 4$ $\alpha\beta = 0$ $\alpha + \beta = -4$ The required quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ $\therefore$ $x^2 - (-4)x + 0 = 0$ $\therefore$ $x^2 + 4x + 0 = 0$ $\therefore$ $\boldsymbol{x^2 + 4x = 0}$	1 1
(iii)	$x^2 - x - 3 = 0$ Comparing the given equation with $ax^2 + bx + c = 0$, we get; $\therefore$ $\boldsymbol{a = 1, b = -1, c = -3}$	 1 1
(iv)	When two coins are tossed the sample space is $\therefore$ S = { HH, HT, TH, TT } $\therefore$ $n(S) = 4$ Let A be the probability of not getting a head. $\therefore$ A = { TT } $\therefore$ $n(A) = 1$ $\therefore$ $\mathbf{P(A)} = \frac{n(A)}{n(S)} = \frac{1}{4}$	 ½ ½ 1

(v) **1**

Turnover (Class interval)	Class Mark (x_i)	No. of stores (Frequency) (f_i)	$f_i x_i$
5 – 15	10	5	50
15 – 25	20	10	200
25 – 35	30	20	600
35 – 45	40	13	520
45 – 55	50	2	100
Total	–	$\sum f_i = 50$	$\sum f_i x_i = 1470$

$\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{1470}{50} = 29.40$ ½

∴ **Mean turnover is I 29.40 lakhs.** ½

(vi) (a) Total sale due to salesman A = Rs. 18000

Measure of central angle $= \dfrac{\text{Sale due to A}}{\text{Total Sale}} \times 360^\circ$

$90^\circ = \dfrac{18000}{\text{Total Sale}} \times 360^\circ$

Total sale $= \dfrac{18000 \times 360^\circ}{90^\circ}$

Total sale = Rs. 72000 ...(i) ½

Total sale due to Salesman B

Measure of central angle $= \dfrac{\text{Sale due to salesman B}}{\text{Total sale}} \times 360^\circ$

$120 = \dfrac{\text{Sale due to salesman B}}{72000} \times 360^\circ$

[Given and from (i)]

∴ Sale due to B $= \dfrac{120 \times 72000}{360^\circ}$

∴ Sale due to B = Rs. 24000 ½

Total sale due to Salesman C

Measure of central angle $= \dfrac{\text{Sale due to Salesman C}}{\text{Total sale}} \times 360^\circ$

$120 = \dfrac{\text{Sale due to Salesman C}}{72000} \times 360^\circ$

[Given and from (i)]

$\therefore$ Sale due to C $= \dfrac{80 \times 72000}{360°}$

Sale due to C $=$ Rs. 16000

Sale due to Salesman D

Sale due to D = Total sale – (A's sale + B's sale + C's sale)

= 72000 – (18000 + 24000 + 16000)

= 72000 – 58000

$\therefore$ Sale due to D = Rs. 14000 ½

$\therefore$ **Salesman B is with the highest sale of Rs. 24000.** ½

A.3. Attempt any four of the following subquestions :

(i) Let a be the first term and d be the common difference.

Here, $S_{55} = 3300$

$S_n = \dfrac{n}{2}[2a + (n-1)d]$ **1**

$\therefore S_{55} = \dfrac{55}{2}[2a + (55-1)d]$

$\therefore 3300 = \dfrac{55}{2}(2a + 54d)$

$\therefore 3300 = \dfrac{55}{2} \times 2(a + 27d)$

$\therefore a + 27d = \dfrac{6600}{55}$

$\therefore a + 27d = 60 \quad \ldots(i)$

$t_n = a + (n-1)d$ **1**

$\therefore t_{28} = a + (28-1)d$

$\therefore t_{28} = a + 27d \quad \ldots(ii)$

$\therefore t_{28} = 60$

$\therefore$ **The 28th term of the given A.P. is 60.** **1**

(ii) $3y^2 + 7y + 1 = 0$

$3y^2 + 7y = -1$

Dividing both sides by 3, we get;

$y^2 + \dfrac{7}{3}y = -\dfrac{1}{3} \quad \ldots (i)$

Third term $= \left(\dfrac{1}{2} \times \text{coefficient of } y\right)^2$ **1**

$$= \left(\frac{1}{2} \times \frac{7}{3}\right)^2$$

$$= \left(\frac{7}{6}\right)^2$$

$$= \frac{49}{36}$$

Adding $\frac{49}{36}$ to both sides of eq. (i), we get;

$$y^2 + \frac{7}{3}y + \frac{49}{36} = -\frac{1}{3} + \frac{49}{36}$$

$$\therefore \quad \left(y + \frac{7}{6}\right)^2 = \frac{-12 + 49}{36}$$

$$\therefore \quad \left(y + \frac{7}{6}\right)^2 = \frac{37}{36}$$ **1**

Taking square root on both the sides, we get;

$$y + \frac{7}{6} = \frac{\pm\sqrt{37}}{6}$$

$$\therefore \quad y = \frac{-7}{6} \pm \frac{\sqrt{37}}{6}$$

$$\therefore \quad y = \frac{-7 \pm \sqrt{37}}{6}$$

$$\therefore \quad y = \frac{-7 + \sqrt{37}}{6} \quad \text{or} \quad y = \frac{-7 - \sqrt{37}}{6}$$

$\therefore$ $\mathbf{\frac{-7 + \sqrt{37}}{6}}$ **and** $\mathbf{\frac{-7 - \sqrt{37}}{6}}$ **are roots of the given quadratic equation.** **1**

(iii) $4x + y = 7$; $16x + ky = 28$

Comparing the above equations with the equations

$a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, we get,

$a_1 = 4, \quad b_1 = 1, \quad c_1 = 7$

$a_2 = 16, \quad b_2 = k, \quad c_2 = 28$

	$\frac{a_1}{a_2} = \frac{4}{16} = \frac{1}{4}$, $\frac{b_1}{b_2} = \frac{1}{k}$ and $\frac{c_1}{c_2} = \frac{7}{28} = \frac{1}{4}$	**1**
	The condition for simultaneous equations to have infinitely many solutions is	
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	**1**
	$\therefore \frac{1}{4} = \frac{1}{k} = \frac{1}{4}$	
	$\therefore \frac{1}{4} = \frac{1}{k}$	
	$\therefore$ $\boldsymbol{k = 4}$	**1**
(iv)	$x = 5$ and $y = 3$ is the solution of $3x + ky = 3$	**1**
	$\therefore x = 5$ and $y = 3$ should satisfy the equation $3x + ky = 3$	
	Substituting $x = 5$ and $y = 3$ in the equation $3x + ky = 3$, we get;	
	$3(5) + k(3) = 3$	
	$\therefore 15 + 3k = 3$	
	$\therefore 3k = 3 - 15$	**1**
	$\therefore 3k = -12$	
	$\therefore k = \frac{-12}{3}$	
	$\therefore$ $\boldsymbol{k = -4}$	**1**
(v)	Total number of students = 100	
	$\therefore n(S) = 100$	
	Let A be the event that students drink tea	
	$\therefore n(A) = 60$	
	$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{60}{100} = \frac{3}{5}$	**1**
	Let B be the event that students drink coffee	
	$\therefore n(B) = 50$	
	$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{50}{100} = \frac{1}{2}$	**1**
	Let $A \cup B$ be the event that students drink both tea and coffee.	

	$\therefore$ $n(\text{A} \cap \text{B}) = 30$	
	$\therefore$ $\text{P}(\text{A} \cap \text{B}) = \dfrac{n(\text{A} \cap \text{B})}{n(\text{S})} = \dfrac{30}{100} = \dfrac{3}{10}$	
	By Addition theorem of probability,	
	$\text{P}(\text{A} \cup \text{B}) = \text{P}(\text{A}) + \text{P}(\text{B}) - \text{P}(\text{A} \cap \text{B})$	
	$= \dfrac{3}{5} + \dfrac{1}{2} - \dfrac{3}{10}$	
	$= \dfrac{6 + 5 - 3}{10}$	
	$= \dfrac{8}{10}$	
	$\therefore$ $\mathbf{P(A \cup B) = \dfrac{4}{5}}$	**1**
A.4.	**Attempt any three of the following subquestions :**	
(i)	Let 'a' be the first term and 'd' be the common difference of A.P.	
	We conveniently choose three consecutive terms of A.P.	
	as $a - d$, a, $a + d$	
	The sum of these terms is -3,	
	$\therefore$ $(a - d) + a + (a + d) = -3$	
	$\therefore$ $3a = -3$	
	$\therefore$ $a = -1$	**1**
	Product of their cubes is 512	
	$\therefore$ $(a - d)^3 \times (a)^3 \times (a + d)^3 = 512$	
	$\therefore$ $[(a - d) \times a \times (a + d)]^3 = 512$	
	$\therefore$ $a^3[(a - d)\ (a + d)]^3 = 512$	
	$\therefore$ $a^3(a^2 - d^2)^3 = 512$	
	Taking cube roots on both sides, we get;	
	$a(a^2 - d^2) = 8$	**1**
	Substituting $a = -1$, we get;	
	$-1\ (1 - d^2) = 8$	
	$\therefore$ $-1 + d^2 = 8$	
	$\therefore$ $d^2 = 9$	
	$\therefore$ $d = 3$	**1**
	$\therefore$ The three terms are	
	$a - d = -1 - 3 = -4$	
	$a = -1$	
	$a + d = -1 + 3 = 2$	
	$\therefore$ **Three consecutive terms in A.P. are –4, –1, 2.**	**1**

(ii)	Let the roots of the quadratic equation be α and β	
	We have, $\alpha - \beta = 5$ ...(i)	
	and $\alpha^3 - \beta^3 = 215$ ...(ii)	
	$\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$	
	$\therefore\ 215 = (5)^3 + 3\alpha\beta(5)$	1
	[From equation (i) and (ii)]	
	$\therefore\ 215 = 125 + 15\alpha\beta$	
	$\therefore\ 215 - 125 = 15\alpha\beta$	
	$\therefore\ 15\alpha\beta = 90$	
	$\therefore\ \alpha\beta = \frac{90}{15}$	
	$\therefore\ \alpha\beta = 6$	1
	Now,	
	$(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$	
	$\therefore\ (\alpha + \beta)^2 = (5)^2 + 4 \times 6$	
	$\therefore\ (\alpha + \beta)^2 = 25 + 24$	
	$\therefore\ (\alpha + \beta)^2 = 49$	
	$\therefore\ (\alpha + \beta) = \pm\sqrt{49}$ [Taking square roots]	
	$\therefore\ (\alpha + \beta) = \pm 7$	1
	$\therefore\ \alpha + \beta = +7$ or $\alpha + \beta = -7$	
	The required quadratic equation is	
	$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$	
	$\therefore\ x^2 - (7x) + 6 = 0$ or $x^2 - (-7)x + 6 = 0$	
	$\therefore\ x^2 - 7x + 6 = 0$ or $x^2 + 7x + 6 = 0$	
	$\therefore$ **The required quadratic equation is $x^2 - 7x + 6 = 0$ or $x^2 + 7x + 6 = 0$**	1
(iii)	$2x + 5y = 13$ ...(i)	
	$4x - 9y = 7$ ...(ii)	
	Multiplying eq. (i) by 2, we get;	
	$4x + 10y = 26$ ...(iii)	1
	Subtracting eq. (ii) from eq. (iii), we get;	
	$4x + 10y = 26$	
	$4x - 9y = 7$	
	(–) (+) (–)	
	$19y = 19$	
	$\therefore\ y = \frac{19}{19}$	
	$\therefore\ y = 1$	1

Substituting $y = 1$ in eq. (i), we get;

$2x + 5(1) = 13$

$\therefore$ $2x + 5 = 13$

$\therefore$ $2x = 13 - 5$

$\therefore$ $2x = 8$

$\therefore$ $x = 4$ **1**

$\therefore$ **The point of intersection of the lines $2x + 5y = 13$ and $4x - 9y = 7$ is (4, 1).** **1**

(iv) Two fair dice thrown,

$\therefore$ S = { (1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6) }

$\therefore$ $n(S) = 36$ **½**

i) Let A be the event that sum of points on uppermost faces is perfect square.

$\therefore$ A = { (1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3) }

$\therefore$ $n(A) = 7$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{7}{36}$

Let B be the event that sum of points is divisible by 4

$\therefore$ B = { (1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6) }

$\therefore$ $n(B) = 9$

$\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{9}{36} = \frac{1}{4}$ **½**

$\therefore$ $A \cap B$ = { (1, 3), (2, 2), (3, 1) }

$\therefore$ $n(A \cap B) = 3$

$\therefore$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$ **½**

By Addition theorem of Probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{36} + \frac{1}{4} - \frac{1}{12}$

	$= \dfrac{7+9-3}{36}$	
	$\therefore$ P(A $\cup$ B) $= \dfrac{13}{36}$	½
	ii) Let C be the event that sum is greater than 10.	
	$\therefore$ C = { (5, 6), (6, 5), (6, 6) }	
	$\therefore$ n(C) = 3	
	$\therefore$ P(C) $= \dfrac{n(C)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$	½
	Let D be the event that sum is an odd number.	
	$\therefore$ D = { (1, 2), (2, 1), (3, 2), (4, 1), (5, 2), (6, 1), (1, 4), (2, 3), (3, 4), (4, 3), (5, 3), (6, 3), (1, 6), (2, 5), (3, 6), (4, 5), (5, 5), (6, 5) }	
	$\therefore$ n(D) = 18	
	$\therefore$ P(D) $= \dfrac{n(D)}{n(S)} = \dfrac{18}{36} = \dfrac{1}{2}$	½
	$\therefore$ C $\cap$ D = { (5, 6), (6, 5) }	
	$\therefore$ n(C $\cap$ D) = 2	
	$\therefore$ P(C $\cap$ D) $= \dfrac{n(C \cap D)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$	½
	By Addition theorem of probability,	
	P(C $\cup$ D) = P(C) + P(D) − P(C $\cap$ D)	
	$= \dfrac{1}{12} + \dfrac{1}{2} - \dfrac{1}{18}$	
	$= \dfrac{3+18-2}{36}$	
	$\therefore$ **P(C $\cup$ D) = $\mathbf{\dfrac{19}{36}}$**	½
A.5.	**Attempt any four of the following subquestions :**	
(i)	Let the two numbers be x and y.	
	A.M. (A) $= \dfrac{x+y}{2}$	
	G.M. (G) $= \sqrt{xy}$	
	Given $\dfrac{A}{G} = \dfrac{5}{4}$	
	$\therefore$ $\dfrac{\frac{x+y}{2}}{\sqrt{xy}} = \dfrac{5}{4}$	**1**

$\therefore \quad 4\left(\dfrac{x+y}{2}\right) = 5\sqrt{xy}$

$\therefore \quad 2(x+y) = 5\sqrt{xy}$ **1**

Squaring both sides, we get;

$4(x+y)^2 = 25xy$

$\therefore \quad 4(x^2 + 2xy + y^2) = 25xy$

$\therefore \quad 4x^2 + 8xy + 4y^2 - 25xy = 0$

$\therefore \quad 4x^2 - 17xy + 4y^2 = 0$

$\therefore \quad 4x^2 - 16xy - xy + 4y^2 = 0$

$\therefore \quad 4x(x - 4y) - y(x - 4y) = 0$

$\therefore \quad (x - 4y)(4x - y) = 0$

$\therefore \quad x - 4y = 0$ or $4x - y = 0$

$\therefore \quad x = 4y$ or $y = 4x$

But $x + y = 30$ ...(i) [Given] **1**

Substituting $x = 4y$, in eq.(i) we get;

$4y + y = 30$

$\therefore \quad 5y = 30$

$\therefore \quad y = 6$ **½**

When $y = 6$,

$x + 6 = 30$

$x = 30 - 6$

$x = 24$ **½**

$\therefore$ The two numbers are 24 and 6.

Substituting $y = 4x$ in eq.(i), we get;

$\therefore \quad x + 4x = 30$

$\therefore \quad 5x = 30$

$\therefore$ when $x = 6$

$\therefore \quad 6 + y = 30$

$\therefore \quad y = 24$

$\therefore \quad x = 6,\ y = 24.$

$\therefore$ **The two numbers are 6 and 24.** **1**

(ii) $2x = y + 2$ $\qquad$ $4x + 3y = 24$

$\therefore \quad y = 2x - 2$ $\qquad$ $3y = -4x + 24$

$y = \dfrac{-4x + 24}{3}$

x	2	−1	−2
y	2	−4	−6
(x, y)	(2, 2)	(−1, −4)	(−2, −6)

x	1.5	3	0
y	6	4	8
(x, y)	(1.5, 6)	(3, 4)	(0, 8)

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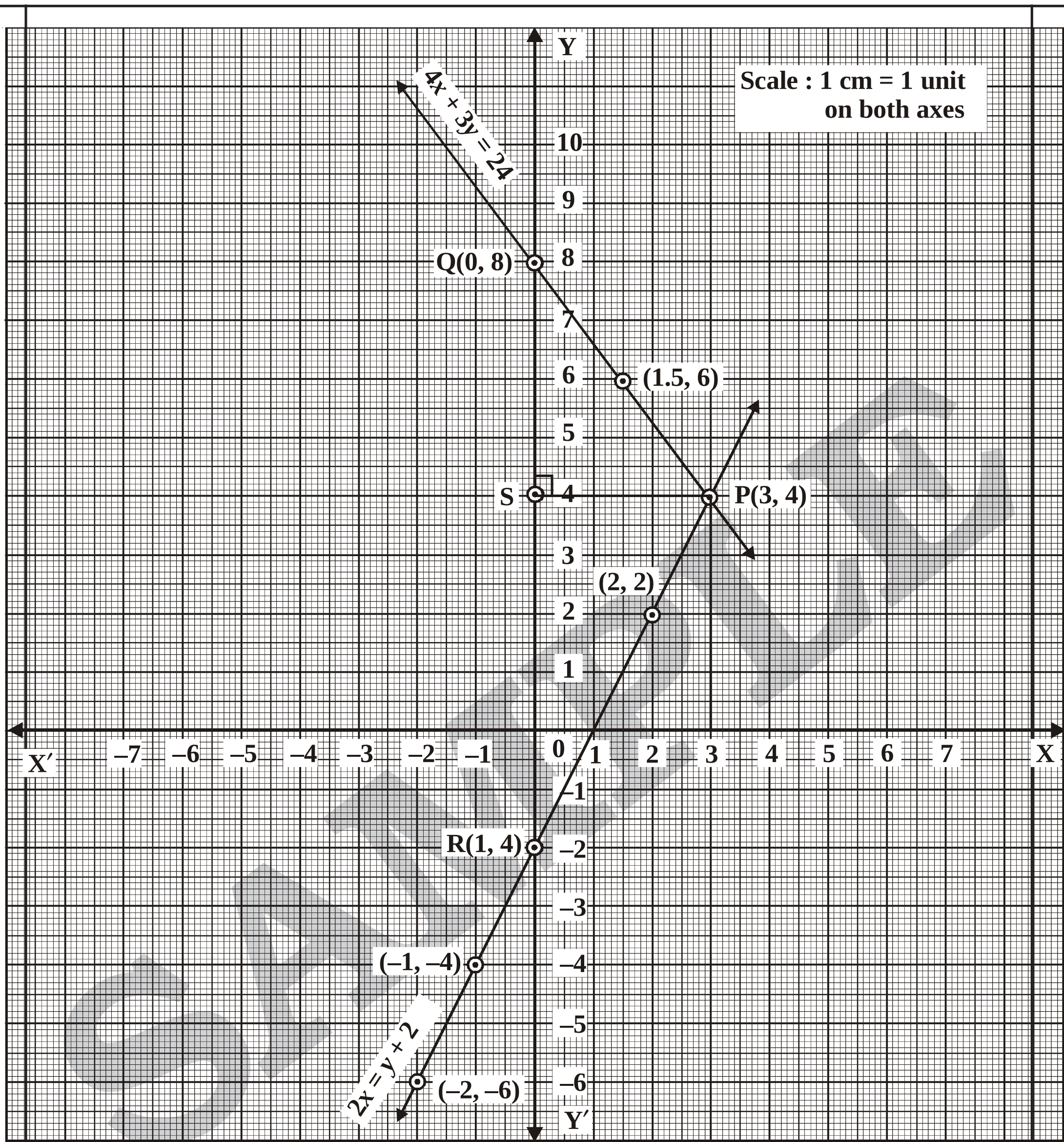

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$\therefore$ **The solution of the given simultaneous equation is (3, 4)** 1

$$A(\triangle PQR) = \frac{1}{2} \times PS \times QR$$

$$= \frac{1}{2} \times 3 \times 10$$

= 15 sq. units. 1

(iii) Let the weight of the empty bucket be x kg and the weight of the water when it is completely filled be y kg.
From the first condition, we get,

$$x + \frac{3}{5}y = 15 \qquad \ldots\text{(i)}$$ **1**

From the second condition, we get;

$$x + \frac{4}{5}y = 19 \qquad \ldots\text{(ii)}$$ **1**

Subtracting eq. (i) from (ii), we get;

$$x + \frac{4}{5}y = 19$$

$$x + \frac{3}{5}y = 15$$

$$(-) \quad (-) \quad (-)$$

$$\frac{1}{5}y = 4$$

$\therefore$ $y = 4 \times 5 = 20$ **1**

Substituting $y = 20$ in eq. (i), we get;

$$x + \frac{3}{5} \times 20 = 15$$

$\therefore$ $x + 12 = 15$

$\therefore$ $x = 15 - 12 = 3$ **1**

$\therefore$ The weight of the empty bucket is 3 kg and the weight of the water when it is completely filled is 20 kg.

$\therefore$ Weight of the bucket when it is filled completely $= x + y$

$= 3 + 20$

$= 23$ kg

$\therefore$ **Weight of the bucket is 23 kg when it is filled completely.** **1**

(iv) Here, A = 37.5, h = 5

Duration (in sec) Class intervals	Class Mark (x_i)	$d_i = x_i - \text{A}$ $= x_i - 37.5$	$u_i = \frac{d_i}{h}$ $= \frac{d_i}{5}$	No. of advertisement	$f_i u_i$
25 – 30	27.5	–10	–2	10	–20
30 – 35	32.5	–5	–1	32	–32
35 – 40	37.5 → A	0	0	15	0
40 – 45	42.5	5	1	9	9
45 – 50	47.5	10	2	7	14
50 – 55	52.5	15	3	2	6
				$\sum f_i = 75$	$\sum f_i u_i$ –23

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$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

$$= \frac{-23}{75} = -0.307$$ **1**

Mean $(\bar{x}) = A + h\bar{u}$

$= 37.5 + 5 \times (-0.307)$

$= 37.5 - 1.535 = 35.965$

$\therefore$ **The mean duration of advertisement on T.V. is 35.97 seconds.** **1**

(v) Since total percentage is 100

$15 + 20 + a + a + 25 = 100$

$\therefore\ 2a + 60 = 100$

$\therefore\ 2a = 100 - 60$

$\therefore\ 2a = 40$

$\therefore\ a = 20$

$\therefore$ Percentage of population of Russia and USA = 20% each. **1**

Country	Percentage of world population	Central angle
India	15	$\frac{15}{100} \times 360 = 54°$
China	20	$\frac{20}{100} \times 360 = 72°$
Russia	20	$\frac{20}{100} \times 360 = 72°$
USA	20	$\frac{20}{100} \times 360 = 72°$
Others	25	$\frac{25}{100} \times 360 = 90°$
Total	100	360°

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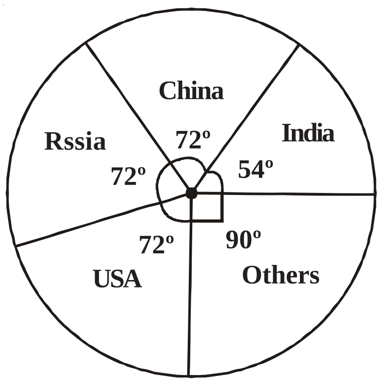

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